

# Lecture 3: Parameter estimation & patient- specific calibration

Paul Macklin, Ph.D.  
Lecturer  
University of Dundee

23 August 2010

# Motivation

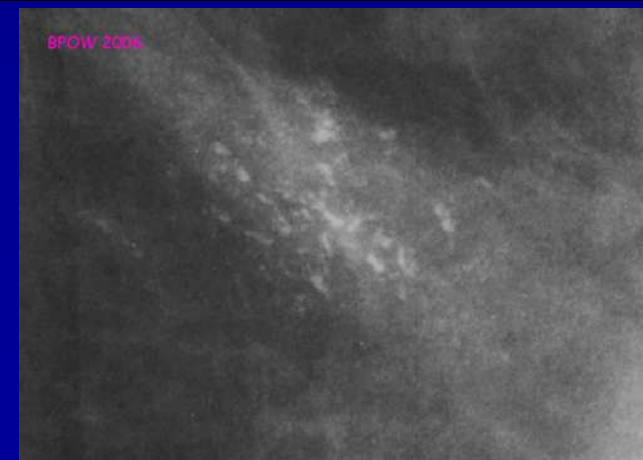
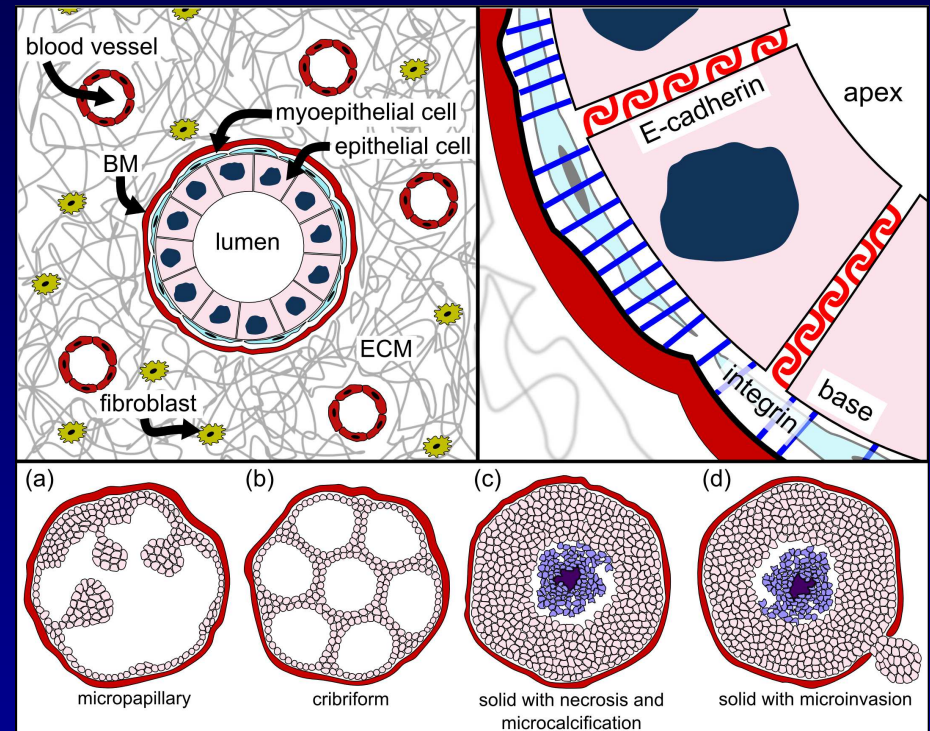
- Want to calibrate to patient data
- Want to make testable predictions on cancer biology
- **Tautology**: Complex models have many parameters. The apparent agreement between model behavior and data may be due to tuning free parameters. (“cheating”)
- To be predictive, we need to constrain the parameters.
  - Try to use simplest model possible with fewer parameters.
  - Estimate from experimental / theoretical biology.
  - Use mathematical analyses to extract more from the data.
  - For patient-specific behaviour, need to incorporate noisy data.
- Ideal: set the correct physical parameter values, and let the physics do the rest.

# Lecture Outline

- Brief introduction to DCIS
- Estimating easy parameters
  - Cell cycle length, G1 length
  - Some oxygen parameters
- Estimating tricky parameters
  - Apoptosis time
  - Necrosis and calcification parameters
  - Mechanics parameters
- What patient data are available?
- Patient-specific calibration
  - Geometric parameters
  - Oxygen transport
  - Population dynamics
  - Cell-cell adhesion mechanics
- Sample Calibration and Verification
- Coming next
- References

# Brief introduction to DCIS

- Ductal carcinoma in situ (DCIS)
- Growth constrained to lumen by BM
- Not yet invasive
- Precursor to invasive carcinoma
- Growth fed by diffusion from outside duct
- Subtypes depend in part upon mechanics, cell polarisation
- Cribriform and micropapillary: cells polarised, but no anoikis
- Solid-type and comedo: cells unpolarised, oxygen diffusion limitations, necrosis
- Microinvasion: Cells break through BM and invade stroma
- Notable feature: calcification of necrotic material – prime detection method by mammogram



# Lecture Outline

- Brief Introduction to DCIS
- Estimating easy parameters
  - Cell cycle length, G1 length
  - Some oxygen parameters
- Estimating tricky parameters
  - Apoptosis time
  - Necrosis and calcification parameters
  - Mechanics parameters
- What patient data are available?
- Patient-specific calibration
  - Geometric parameters
  - Oxygen transport
  - Population dynamics
  - Cell-cell adhesion mechanics
- Sample Calibration and Verification
- Coming next
- References

# Estimating easy parameters: Cell cycle length, G1 length

- Typical time scales are typically 18 to 24 hours
  - Byrne, Owen, Ward, King, others
- We separate  $G_0$  from cycle, so take shorter end
  - $\tau_p = 18$  hours
- Experimental work by Smith & Martin (1973):
  - $S + G_2 + M = 9$  hours ( $= \frac{1}{2} \tau_p$ )
  - $\tau_p$  (their  $T_B$ ) also in the range of 10 to 24 hours

# Lecture Outline

- Brief Introduction to DCIS
- Estimating easy parameters
  - Cell cycle length, G1 length
  - Some oxygen parameters
- Estimating tricky parameters
  - Apoptosis time
  - Necrosis and calcification parameters
  - Mechanics parameters
- What patient data are available?
- Patient-specific calibration
  - Geometric parameters
  - Oxygen transport
  - Population dynamics
  - Cell-cell adhesion mechanics
- Sample Calibration and Verification
- Coming next
- References

# Estimating easy parameters: Some oxygen parameters

- Oxygen diffusion length scale  $L \sim 100 \mu\text{m}$ 
  - Byrne, Owen, Ward, King, others, 1970s experiments ...
  - $D \sim 50 \mu\text{m}^2/\text{s} = 3000 \mu\text{m}^2/\text{min} \leftarrow$  Owen et al. 2004
- Use this and O<sub>2</sub> diffusion coefficient to derive mean O<sub>2</sub> uptake in the viable rim

$$L = \sqrt{\frac{D}{\langle \lambda \rangle}}$$

- Get mean uptake  $\sim 0.1$  1/min
- Compare Ward and King (1997) proliferation rate at the step function limit to our  $\alpha_p$ , and get  $\sigma_H \sim 0.2$



# Lecture Outline

- Brief introduction to DCIS
- Estimating easy parameters
  - Cell cycle length, G1 length
  - Some oxygen parameters
- Estimating tricky parameters
  - Apoptosis time
  - Necrosis and calcification parameters
  - Mechanics parameters
- What patient data are available?
- Patient-specific calibration
  - Geometric parameters
  - Oxygen transport
  - Population dynamics
  - Cell-cell adhesion mechanics
- Sample Calibration and Verification
- Coming next
- References

# Estimating tricky parameters: Apoptosis time

- Can be a difficult process to observe from start to finish
- Not as much data out there as we'd like – clinical interest tends to end at the start of apoptosis
- Order of magnitude estimate: on the order of hours
  - Makes biological / chemical sense: inactivated Caspases are pre-positioned and ready to go
  - Makes intuitive sense: destruction is easier than creation
- Use mathematical model + IHC to estimate
- Lee et al. 2006:
  - measured AI (TUNEL assay) and PI (Ki-67) in non-cancerous breast epithelium
  - Several hundred pre-menopausal women
  - Several hundred post-menopausal women
  - Large enough sample size that menstrual oscillations balance out
  - Assume that tissue is overall in homeostasis, when averaged across the menstrual cycle

# Estimating tricky parameters: Apoptosis time

- Assume (a la Hanahan & Weinberg) that tumour and normal cells use same processes with altered frequency
  - Assume same cell cycle length ~18h
  - same apoptosis length
- Use the volume-averaged estimate for normoxic tissue
  - $N' = PI / \tau_P - AI / \tau_A$
  - Solve for  $\tau_A$
- Get 5.71 hours from pre-menopausal data
- Get 5.62 hours from post-menopausal data
- Get similar results for differing hormonal situations
  - gives credence to the approach

# Estimating tricky parameters: Apoptosis time

- Want to adjust the estimate to account for imaging and detection shortcomings
  - TUNEL assay: detects DNA fragmentation
  - Cleaved Caspase-3: detects an “executioner” protein present through most of apoptosis
  - Both detect apoptosis after initial processes – probably underestimate apoptosis
- Turn to experimental biology on individual cells
- Scarlet et al. 2003: apoptosis in Jurkat cells
  - Induction of apoptosis (“priming phase”):
    - Rapid loss of mitochondrial membrane voltage potential
    - Rapid change in ATP:ADP → visible marker of energy loss
    - Occurs on order of minutes

# Estimating tricky parameters: Apoptosis time

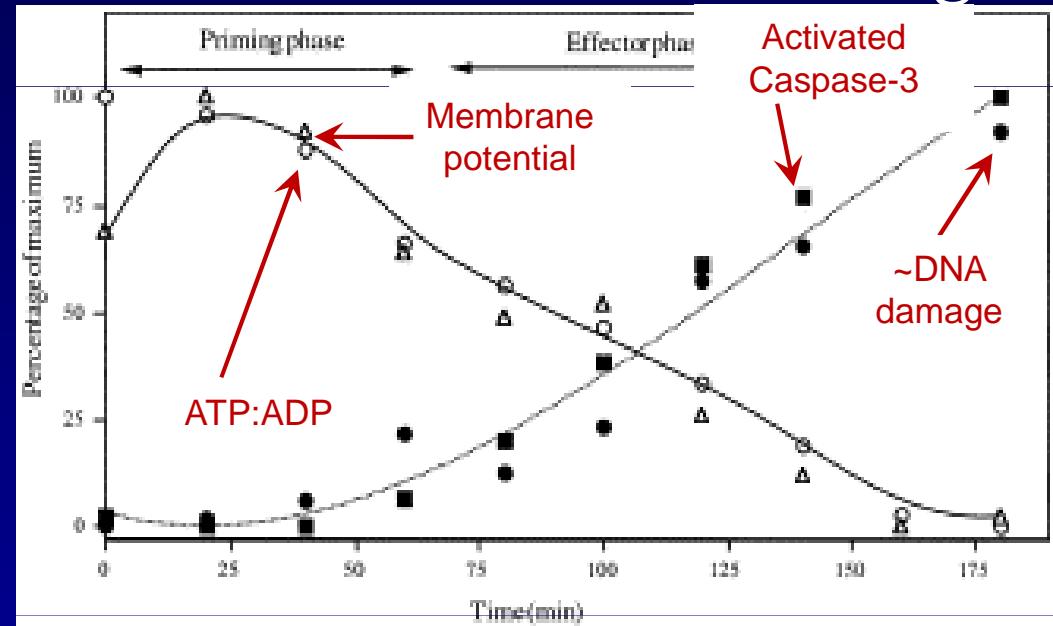
- Scarlet et al. 2003: apoptosis in Jurkat cells – Later during apoptosis:

- Cleaved caspase-3:

- 0-60 min: ~0 detection
- 50-60 min: 10% of peak
- 180 min: peak level
- → misses first 1-2 hours

- TUNEL assay:

- Little DNA laddering for 3 h.



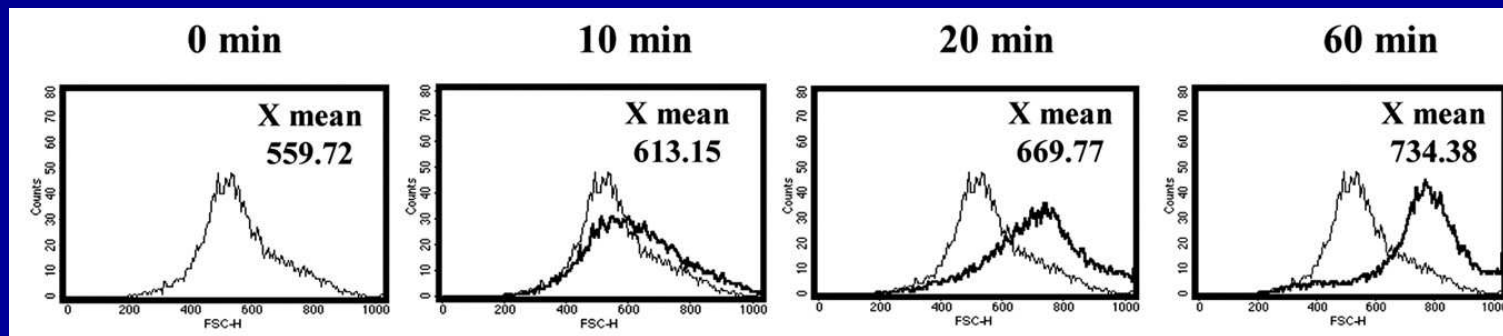
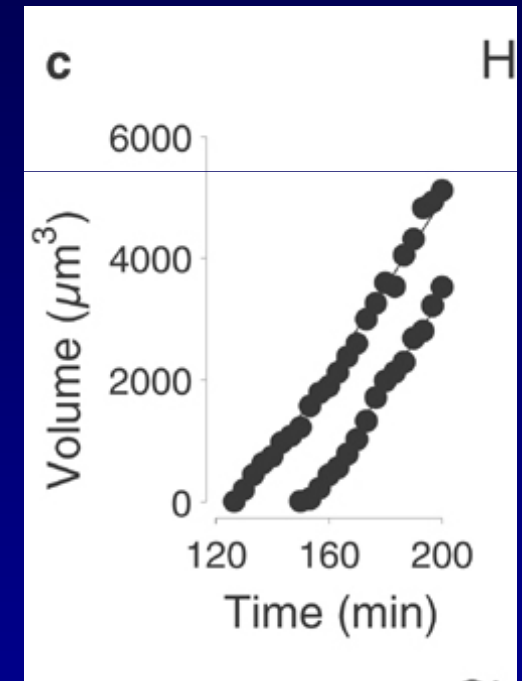
- Adjust estimate to about  $\tau_A = 8.6h$
- This is an example of combining experimental and theoretical biology literature with a mathematical understanding to improve modelling.
- Also shows need to understand the core biology and immunohistochemistry to better constrain the model.

# Lecture Outline

- Brief introduction to DCIS
- Estimating easy parameters
  - Cell cycle length, G1 length
  - Some oxygen parameters
- Estimating tricky parameters
  - Apoptosis time
  - Necrosis and calcification parameters
  - Mechanics parameters
- What patient data are available?
- Patient-specific calibration
  - Geometric parameters
  - Oxygen transport
  - Population dynamics
  - Cell-cell adhesion mechanics
- Sample Calibration and Verification
- Coming next
- References

# Estimating tricky parameters: Necrosis and calcification

- Early swell swelling (and magnitude), lysis
  - Ion pumps lose energy early
  - Ion pumps ordinarily necessary for apoptosis
  - Expect swelling and lysis faster than apoptosis
    - Estimate around 6 hours to swell and lyse
  - Jun et al. 2007: 30% swelling in 60 minutes
  - Barros et al. 2003: necrotic blebs grow linearly, 200 min
  - Grönroos et al. 2005: 1.5-fold increase in 12 h
  - Wu et al. 2010: 2- to 5-fold volume in 24 h.
    - Estimate a conservative  $f_{NS}$  0.3 to 1.0 for start
- Time to lose functional adhesion receptors
  - No easy estimate, but necrotic material seems to aggregate even prior to calcification
  - Some sort of adhesion (possibly not by E-cadherin) on order of days



# Estimating tricky parameters: Necrosis and calcification

- Very little data on calcification
- Difficult to study *in vitro*
- Best data: aortic calcification, heart disease
- Jian et al. 2003: significant calcification in *post mortem* cardiac valves in 7 days (10% increase in Ca) to 14 days (40% increase)
- Lee et al. 2006: gradual elastin calcification in rat subdermal model in 2-3 weeks
- Gadeau et al. 2001: calcified rabbit aorta after balloon angioplasty injury, calcium deposits in 2-4 days, increased through 8 days, approached steady in 8-30 days
- Our estimate: on the order of **weeks**
- Hone estimate by simulation, compare percentage of necrotic core occupied by calcification to comedo DCIS (Macklin et al. 2009)
  - 15-day parameter value gave best match (for older model)
  - Newer model with necrotic volume changes: value may even be higher?



$\tau_C$ (days)	0.5	1	5	15	30
Fraction of core calcified after 30 days (%)	94.0	83.7	51.1	6.9	0.0



# Lecture Outline

- Estimating easy parameters
  - Cell cycle length, G1 length
  - Some oxygen parameters (L and lambda, s\_H)
- Estimating tricky parameters
  - Apoptosis time
  - Necrosis and calcification parameters
  - Mechanics parameters
- What patient data are available?
- Patient-specific calibration
  - Geometric parameters
  - Oxygen transport
  - Population dynamics
  - Cell-cell adhesion mechanics
- Sample Calibration and Verification
- Coming next
- References

# Estimating tricky parameters: Mechanics parameters

- Solid fraction:
  - experimental data from Macknight et al. (1971) +
  - Assumption solid materials are 1-10 denser than H<sub>2</sub>O
  - $\rightarrow V_s \sim 10\%$
- Cell interaction distance:
  - Byers et al. 1995: MCF-7 and MCF-10A deform 50-70% in sheer flow
  - Guck et al. 2005: After 60 second of stress:
    - MCF10 (benign cell line) deform 10.5%
    - MCF7 (adhesive, moderately aggressive) deform 21.4%
    - MCF7 + weakened cytoskeleton: 30.4%
    - MDA-MB-231 (aggressive): 33.7%
    - For DCIS  $\rightarrow$  estimate  $R_{cca} = R_{cba} \sim 1.214 R$
    - Might be larger due to morphological uncertainty, but good start

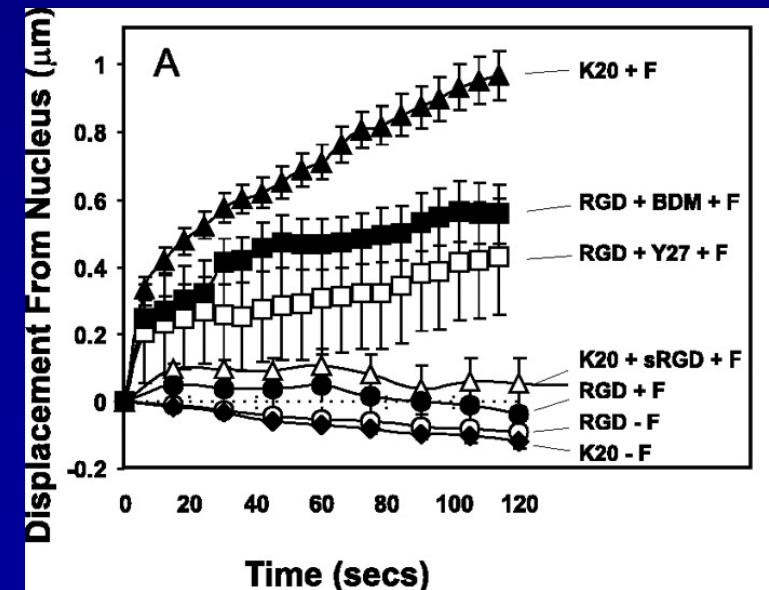
# Estimating tricky parameters: Mechanics parameters

- Cell relaxation: varied time scales (glassy material) Bursac et al. 2005
  - Tenths of seconds: immediate viscoelastic response
  - Minutes, days or more: prolonged stresses
  - Can combine these with magnitude of deformation to estimate  $\alpha_{CCR}/\nu$
  - Matthews et al. 2006: measured motion of magnetic microbeads in cytoskeleton
    - Displacement velocities ( $\alpha_{CCR}/\nu |\nabla \psi|$ ) of 0.1 to 10  $\mu\text{m} / \text{min}$
  - We use  $\alpha_{CCR} = 10 \nu$  to give comparable magnitudes at comparable membrane displacements
  - Just a preliminary estimate

distance from cell center $r$ ( $\mu\text{m}$ )	$\frac{\alpha_{CCR}}{\nu}  \nabla \psi(r) $ ( $\mu\text{m}/\text{min}$ )	distance from cell center $r$ ( $\mu\text{m}$ )	$\frac{\alpha_{CCR}}{\nu}  \nabla \psi(r) $ ( $\mu\text{m}/\text{min}$ )
$R - 0.50 = 9.4530$	0.02524	$R - 0.50 = 9.4530$	0.05047
$R - 1.00 = 8.9530$	0.10095	$R - 1.00 = 8.9530$	0.20189
$R - 2.00 = 7.9530$	0.40379	$R - 2.00 = 7.9530$	0.80757
$R - 3.00 = 6.9530$	0.90852	$R - 3.00 = 6.9530$	1.81704

Table 2

Cell relaxation rate given by  $\nabla \psi$  for  $R = 9.953 \mu\text{m}$ ,  $n_{CCR} = 1$ , and  $\alpha_{CCR}/\nu = 10.00 \mu\text{m}/\text{min}$  (left) and  $20.00 \mu\text{m}/\text{min}$  (right), for small and intermediate deformations. The value of  $M$  does not play a role when  $r > R_N$  (typically 4 to 7  $\mu\text{m}$ ).

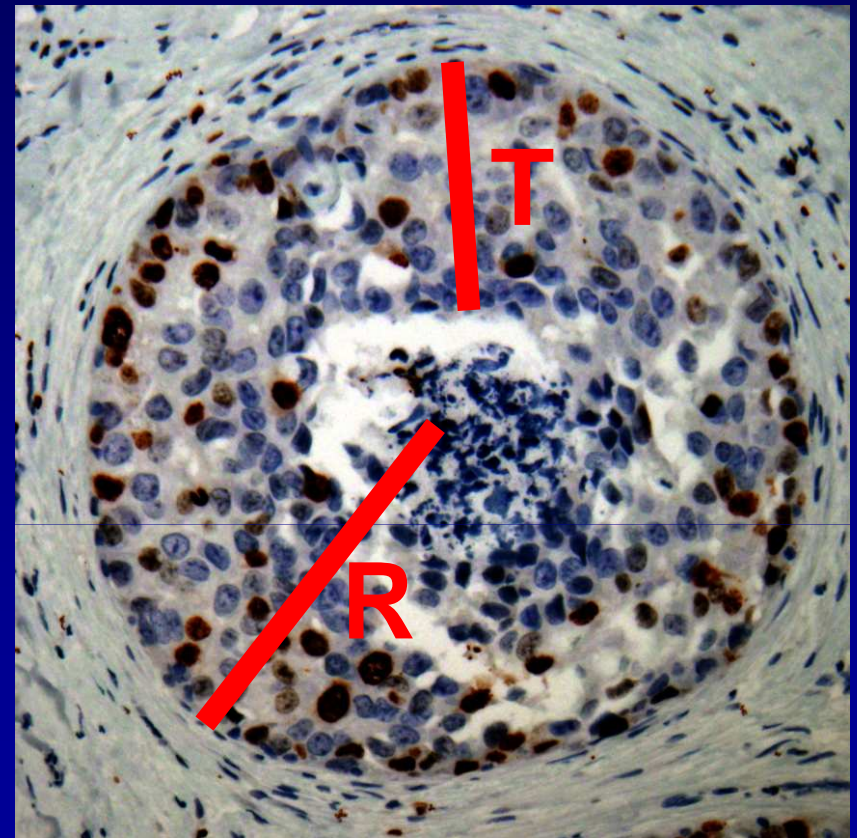


# Lecture Outline

- Brief introduction to DCIS
- Estimating easy parameters
  - Cell cycle length, G1 length
  - Some oxygen parameters
- Estimating tricky parameters
  - Apoptosis time
  - Necrosis and calcification parameters
  - Mechanics parameters
- **What patient data are available?**
- Patient-specific calibration
  - Geometric parameters
  - Oxygen transport
  - Population dynamics
  - Cell-cell adhesion mechanics
- Sample Calibration and Verification
- Coming next
- References

# What patient data are available?

- Histopathology hematoxylin and eosin (H&E)
  - Viable rim thickness ( $T$ )
  - Duct radius ( $R_{\text{duct}}$ )
  - Viable rim confluence ( $f$ ), cell density ( $\rho$ )
  - Tumour microstructure
    - Calcifications
- Immunohistochemistry:
  - Ki-67: proliferation marker : images and PI
  - Nuclear size ( $R_N$ )
  - Cell density, viable rim confluence
  - Cleaved Caspase-3: apoptosis marker : images and AI

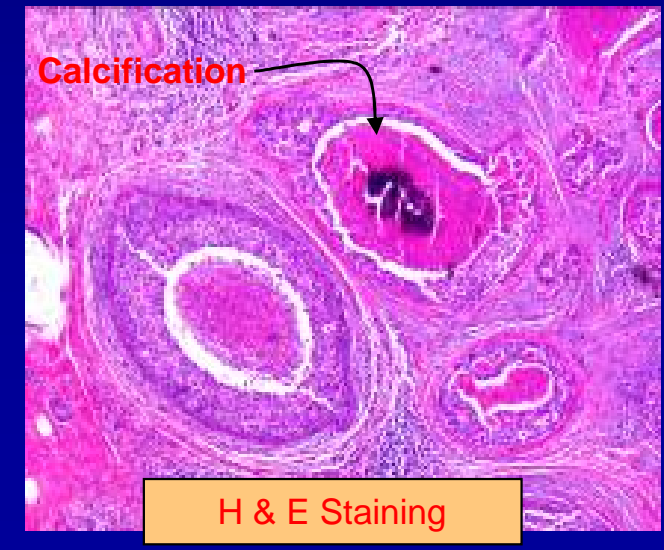
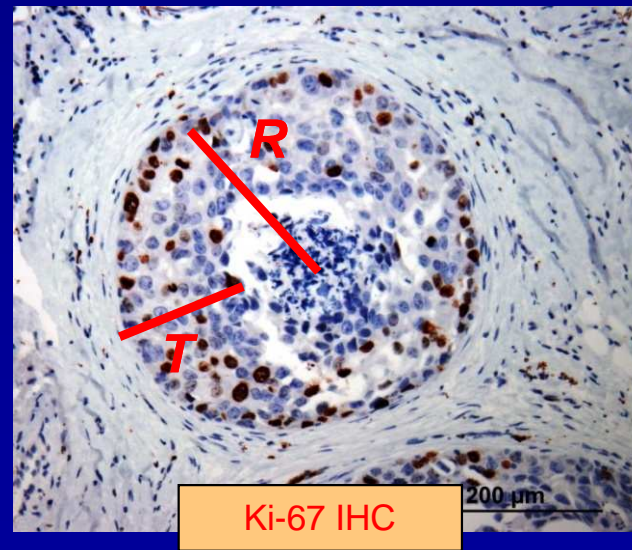
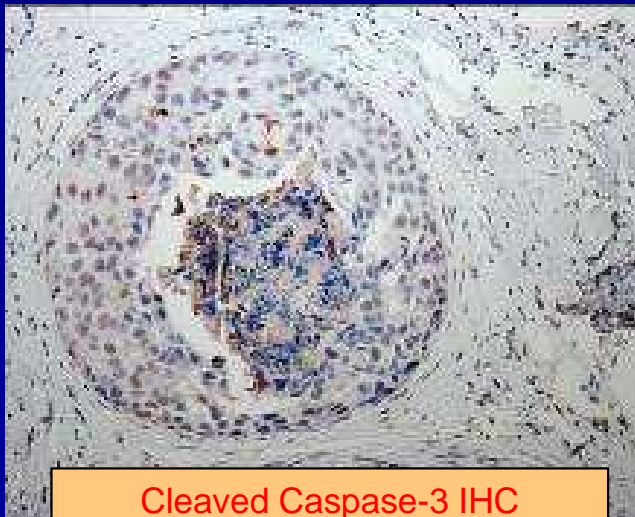


# Data Sources

- **Cleaved Caspase-3 IHC**
  - Often under 1%
  - Terrible stain – cytoplasmic – makes segmentation difficult
  - Must also correct for undercounted cells
- **Measure Proliferative Index (PI) with Ki-67 IHC**
  - Nuclear stain – easier to segment / count
  - Present through most of G<sub>1</sub>, S, G<sub>2</sub>, M
  - Ranges from 5 to 20%

Quantity	Measured Mean	Units
Duct Radius $r_{\text{duct}}$	158.737	$\mu\text{m}$
Viable Rim thickness $T$	78.873	$\mu\text{m}$
PI	17.429	%
raw AI	0.638	%
corrected AI	0.831	%
Cell density $\rho$	0.003217	cells/ $\mu\text{m}^2$

Table 2: Key data for de-identified patient 100019.



# Lecture Outline

- Brief introduction to DCIS
- Estimating easy parameters
  - Cell cycle length, G1 length
  - Some oxygen parameters
- Estimating tricky parameters
  - Apoptosis time
  - Necrosis and calcification parameters
  - Mechanics parameters
- What patient data are available?
- Patient-specific calibration
  - Geometric parameters
  - Oxygen transport
  - Population dynamics
  - Cell-cell adhesion mechanics
- Sample Calibration and Verification
- Coming next
- References

# Patient-Specific Calibration: Geometry

- Relate cell cross-sectional area to density:
  - (confluence) (Area/cell count) =  $f/\rho$  = area of 1 cell
- Relate to spherical approximation to get equivalent radius :
  - $f/\rho = \pi R^2$
- Get duct radius, viable rim thickness, duct geometry from direct measurements



# Lecture Outline

- Brief introduction to DCIS
- Estimating easy parameters
  - Cell cycle length, G1 length
  - Some oxygen parameters
- Estimating tricky parameters
  - Apoptosis time
  - Necrosis and calcification parameters
  - Mechanics parameters
- What patient data are available?
- Patient-specific calibration
  - Geometric parameters
  - Oxygen transport
  - Population dynamics
  - Cell-cell adhesion mechanics
- Sample Calibration and Verification
- Coming next
- References

# Patient-Specific Calibration: Oxygen transport

- Viable rim thickness gives an indirect measurement of oxygen penetration
  - Solve  $O_2$  in idealised cylinder away from tumour leading edge
  - Evaluate to get duct boundary condition

$$\sigma_B = \sigma_H \left[ \cosh \frac{\langle T \rangle}{L} + \sqrt{\Lambda_b} \tanh \left( \frac{\langle R_{\text{duct}} \rangle - \langle T \rangle}{L/\sqrt{\Lambda_b}} \right) \sinh \frac{\langle T \rangle}{L} \right]$$

- Evaluate to get mean oxygen in viable rim

$$\langle \sigma \rangle = \sigma_H \frac{L}{\langle T \rangle} \left[ \sqrt{\Lambda_b} \tanh \left( \frac{\langle R_{\text{duct}} \rangle - \langle T \rangle}{L/\sqrt{\Lambda_b}} \right) \left( \cosh \frac{\langle T \rangle}{L} - 1 \right) + \sinh \frac{\langle T \rangle}{L} \right]$$

- If using different proliferative and non-proliferative uptake rates, apply additional constraints with volume-averaged rate

$$\langle \lambda \rangle = f (\text{PI} \lambda_p + (1 - \text{PI}) \lambda_{np}) + (1 - f) \lambda_b$$

# Lecture Outline

- Brief introduction to DCIS
- Estimating easy parameters
  - Cell cycle length, G1 length
  - Some oxygen parameters
- Estimating tricky parameters
  - Apoptosis time
  - Necrosis and calcification parameters
  - Mechanics parameters
- What patient data are available?
- Patient-specific calibration
  - Geometric parameters
  - Oxygen transport
  - Population dynamics
  - Cell-cell adhesion mechanics
- Sample Calibration and Verification
- Coming next
- References

# Patient-Specific Calibration: Population dynamics

- Four main population dynamic parameters in the viable rim governing  $\mathcal{P} \leftrightarrow \mathcal{Q} \rightarrow \mathcal{A}$  network
- We have AI and PI measurements from IHC
- We estimated the cycle time and apoptosis time
- Assume steady-state population dynamic as in the preceding analysis:
  - We can fully determine the parameters
  - No need to estimate time derivatives from noisy data – temporal information already given by  $\tau_A, \tau_P$
  - Tie to the functional form for  $\alpha_p$  + oxygen estimate to finish

$$\langle \alpha_P \rangle = \frac{\frac{1}{\tau_P} (PI + PI^2) - \frac{1}{\tau_A} AI \cdot PI}{1 - AI - PI}$$
$$\alpha_A = \frac{\frac{1}{\tau_A} (AI - AI^2) + \frac{1}{\tau_P} AI \cdot PI}{1 - AI - PI}.$$

$$\langle \alpha_P \rangle = \bar{\alpha}_P \frac{\langle \sigma \rangle - \sigma_H}{1 - \sigma_H}$$

# Lecture Outline

- Brief introduction to DCIS
- Estimating easy parameters
  - Cell cycle length, G1 length
  - Some oxygen parameters
- Estimating tricky parameters
  - Apoptosis time
  - Necrosis and calcification parameters
  - Mechanics parameters
- What patient data are available?
- Patient-specific calibration
  - Geometric parameters
  - Oxygen transport
  - Population dynamics
  - Cell-cell adhesion mechanics
- Sample Calibration and Verification
- Coming next
- References

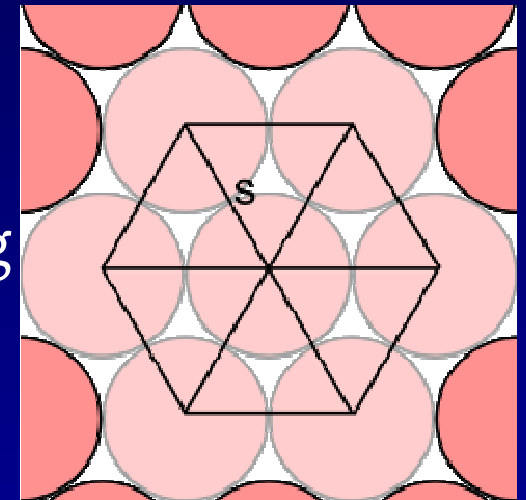
# Patient-Specific Calibration: Cell mechanics

- Density is an indicator of the balance of adhesive and repulsive forces
- Use hexagonal cell packing approximation to determine equilibrium cell spacing

$$s = \sqrt{\frac{2}{\sqrt{3}\langle\rho\rangle}}$$

- Set repulsive and adhesive forces equal at this spacing

$$\varepsilon_i \varepsilon_j \frac{\alpha_{cca}}{\alpha_{ccr}} = \left| \frac{\frac{\partial}{\partial r} \psi \left( s; R_N^i + R_N^j, R_{cca}^i + R_{cca}^j, M, n_{ccr} \right)}{\frac{\partial}{\partial r} \varphi \left( s; R^i + R^j, R_{cca}^i + R_{cca}^j, n_{cca} \right)} \right|$$



- Two coefficient parameters, and we already estimated repulsion → system is determined
- Use tumour microstructure (H&E images) to try to estimate cell-wall mechanics
  - Equilibrium cell-wall distance could be used for similar analysis
  - Without additional forces, need higher cell-wall adhesion for attachment

# Lecture Outline

- Brief introduction to DCIS
- Estimating easy parameters
  - Cell cycle length, G1 length
  - Some oxygen parameters
- Estimating tricky parameters
  - Apoptosis time
  - Necrosis and calcification parameters
  - Mechanics parameters
- What patient data are available?
- Patient-specific calibration
  - Geometric parameters
  - Oxygen transport
  - Population dynamics
  - Cell-cell adhesion mechanics
- **Sample Calibration & Verification**
- Coming next
- References

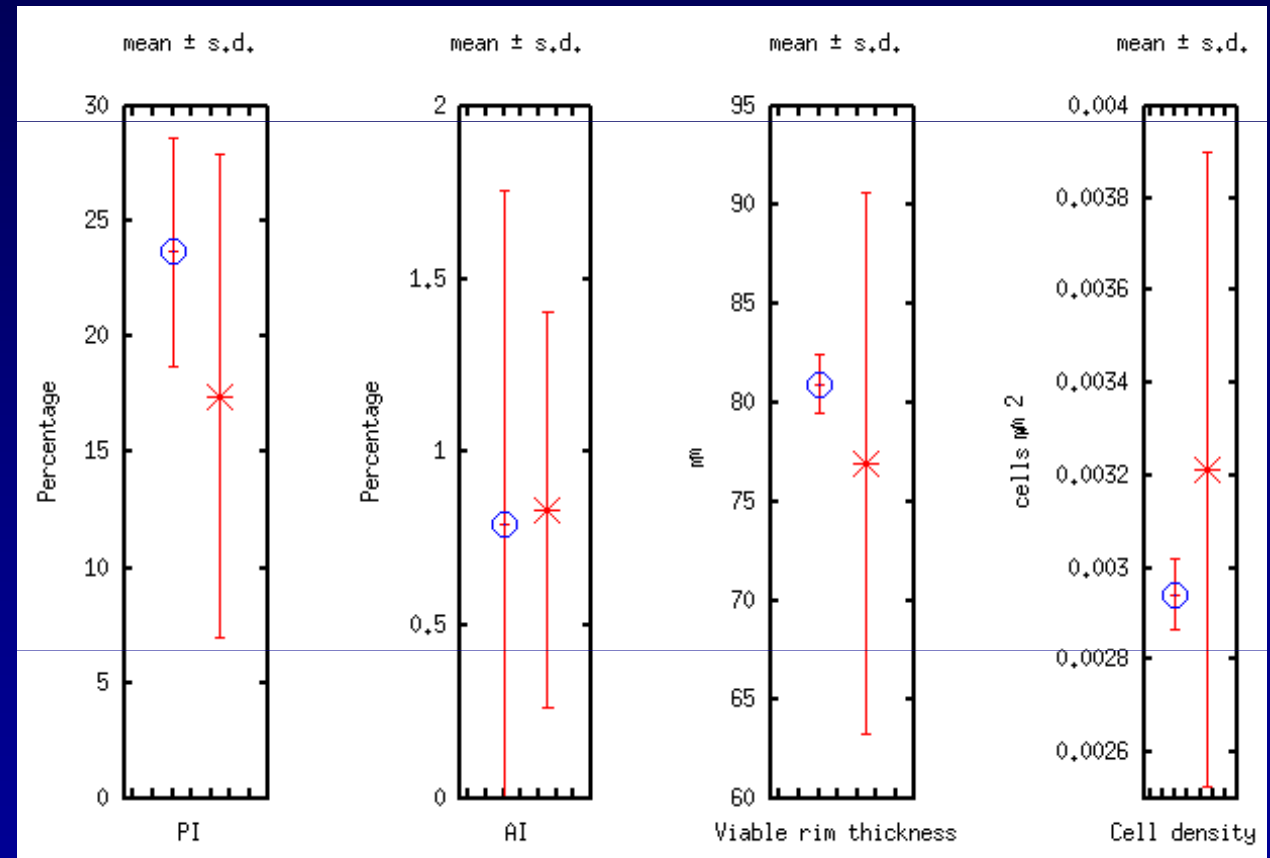
# Sample Calibration and Verification

- Apply the calibration with the following data:
- Run the simulation for 30 simulated days in a 1 mm segment of duct
- Calculate AI, PI, density, and viable rim thickness once per hour
  - Calculate simulated mean and standard deviation
    - Simulated s.d. mostly shows stochasticity, and should decrease as cell number increases
    - But it's also indicative of the heterogeneity across the simulated tumour
  - Restrict away from simulation edges to best match the calibration conditions
  - Compare to mean +/- standard deviation of patient measurements
  - If simulated mean is within [mean - s.d., mean + s.d.], we consider the calibration a “success” (consistent with the data)



# Sample Calibration and Verification: Did we hit our targets?

- PI and AI are within the ranges
- Density is within the range
- Viable rim thickness is within the range
- Perfectionism / Learning:
  - PI a bit high
  - V.R. a bit thick
  - Density a bit low
- Likely scenario:
  - Overestimated confluence
  - Underestimated adhesion strength
  - Underestimated cell uptake rate
  - → increased mean O<sub>2</sub>
  - → larger viable rim
  - → increased proliferation
- Problem is sensitive to O<sub>2</sub>, which interacts with density
- Next time, better measurements of confluence



Blue / Left on each series: simulation  
Red / right on each series: patient data

# Sample Calibration and Verification: Any better with simpler O<sub>2</sub>?

- Do the same calculations with no difference in o<sub>2</sub> uptake rate, to eliminate
- PI and AI are within the ranges
- Density is within the range
- Viable rim thickness is within the range
- All are a better fit, when the dependence of O<sub>2</sub> upon density is eliminated
  - → method is okay, and highlights importance of confluence measurement

All figures given as mean  $\pm$  standard deviation

Quantity	Patient Data	Simulated
PI (%)	17.43 $\pm$ 10.48	17.193 $\pm$ 7.216
AI (%)	0.831 $\pm$ 0.572	1.447 $\pm$ 3.680
Viable rim thickness ( $\mu\text{m}$ )	76.92 $\pm$ 13.70	80.615 $\pm$ 4.454
Cell density (cells/ $\mu\text{m}^2$ )	0.003213 $\pm$ 6.89e-4	0.003336

# Sample Calibration and Verification: A Caution

- Don't get too excited yet. This *isn't* a prediction.
  - It's a validation that your programming and calibration are correct / self-consistent.
- You need to predict something outside your calibration data and model assumptions (e.g., predict behaviour in a different geometry) to claim predictivity.

# Lecture Outline

- Estimating easy parameters
  - Cell cycle length, G1 length
  - Some oxygen parameters
- Estimating tricky parameters
  - Apoptosis time
  - Necrosis and calcification parameters
  - Mechanics parameters
- What patient data are available?
- Patient-specific calibration
  - Geometric parameters
  - Oxygen transport
  - Population dynamics
  - Cell-cell adhesion mechanics
- Sample calibration
- What's next
- References

# Coming Next:

- **Lecture 1:**
  - Cancer biology for modellers
- **Lecture 2:**
  - An agent-based cell model; application to DCIS
- **Lecture 3:**
  - Parameter estimation, patient-specific calibration
- **Lecture 4:**
  - Numerical method, simulation results

# Lecture Outline

- Estimating easy parameters
  - Cell cycle length, G1 length
  - Some oxygen parameters (L and lambda, s\_H)
- Estimating tricky parameters
  - Apoptosis time
  - Necrosis and calcification parameters
  - Mechanics parameters
- What patient data are available?
- Patient-specific calibration
  - Geometric parameters
  - Oxygen transport
  - Population dynamics
  - Cell-cell adhesion mechanics
- Sample calibration
- What's next
- **References**

# Some References

- The calibration presented here is published in:
  - P. Macklin et al. Agent-based modeling of ductal carcinoma in situ: Application to patient-specific breast cancer modeling. In: T. Pham (ed). *Computational Biology: Issues and Applications in Oncology*. Springer, New York, NY USA, 2009. Chapter 4, pages 77-112. ISBN 978-1-4419-0810-0.
  - P. Macklin et al. Agent-based cell modeling: application to breast cancer. In: V. Cristini and J. Lowengrub. *Multiscale Modeling of Cancer*. Cambridge University Press, Cambridge, UK, 2010. Chapter 10, pages 216-44. ISBN 978-0521884426. (in press)
  - P. Macklin et al. A composite agent-based cell model, with application to breast cancer-II: Calibration, Numerical Method and Simulation Results. *J. Theor. Biol.* 2010. (in review)
- Some great DCIS calibration work:
  - See references in publications above by:
    - Owen, Ward, King, Byrne and colleagues: cell energetics, transport
    - Smallbone, Gatenby and colleagues: cell energetics, hypoxia, glycolysis, acidosis

# Contact Information:

- **Email:**

- [macklin@maths.dundee.ac.uk](mailto:macklin@maths.dundee.ac.uk)

- **Web:**

- <http://www.maths.dundee.ac.uk/macklin>

- (new but under construction)

- <http://biomathematics.shis.uth.tmc.edu>

- (old but already built)